International Journal of Civil Engineering and Technology (IJCIET)
Volume 11, Issue 2, February 2020, pp. 183-191, Article ID: IJCIET_11_02_018
Available online at http://www.iaeme.com/ijciet/issues.asp?JType=IJCIET&VType=11&IType=2
ISSN Print: 0976-6308 and ISSN Online: 0976-6316

MODIFICATIONS MODELING OF THE FRIEDLANDER'S BLAST WAVE EQUATION USING THE 6TH ORDER OF POLYNOMIAL EQUATIONS

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ABSTRACT

The rapid release of energy characterizes an explosion as a mass of reactive material is converted into an extremely dense region of high-pressure gas. The gas rapidly expands and displaces the surrounding air, causing a pressure disturbance, which is a blast wave, to propagate away from the center of the explosive at supersonic speed. As the blast wave travels through the air, the front of the pressure disturbance 'shocks up,' the result of a near-discontinuous increase in pressure, density, and temperature. Following the shock front, the pressure decays until ambient pressure is reached, the duration of which is known as the positive phase duration. The semi-empirical 'Friedlander modified equation can describe the positive phase of the blast load reaction on a structure'. Alisjahbana states in a journal that the effect of the explosion load on orthotropic plates is in the post-explosion or post-negative position, which is most influential on the magnitude of the deflection. The 6th order polynomial equation is the best-obtained that corresponds to a small error factor to Normative Equation (Positive Phase without negative phase), Negative Bilinear, Extended Friedlander, Extended Friedlander Teich, Linear-Cubic, Granström negative and positive phase triangle, and Reed's 4th order of polynomial equations. Follow the equations, and do multiply the values of $p_{r,\,max,}$ and $p_{r,\,min}$, to do the final result of the Friedlander modification explosion equation.

Keywords: Friedlander, modified, equation, 6th order, Polynomial.

Cite this Article: H. K. Buwono, S. W. Alisjahbana, Najid, Modifications Modeling of the Friedlander's Blast Wave Equation Using the 6th Order of Polynomial Equations. *International Journal of Civil Engineering and Technology,* 11(2), 2020, pp. 183-191.

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1. INTRODUCTION

1.1. Background

The rapid release of energy characterizes an explosion as a mass of reactive material is converted into an extremely dense region of high-pressure gas. The gas rapidly expands and displaces the surrounding air, causing a pressure disturbance, which a blast wave, to propagate away from the center of the explosive at supersonic speed. As the blast wave travels through the air, the front of the pressure disturbance 'shocks up,' resulting in a near-discontinuous increase in pressure, density, and temperature. Following the shock front, the pressure decays until ambient pressure is reached, the duration of which is known as the positive phase duration. The positive phase of the blast load acting on a structure can be described by the semi-empirical 'modified Friedlander equation' [1]

$$\mathbf{P_r(t) = P_{r,max}\left(1 - \frac{t}{t_d}\right) e^{-bt/td}} \qquad (1)$$

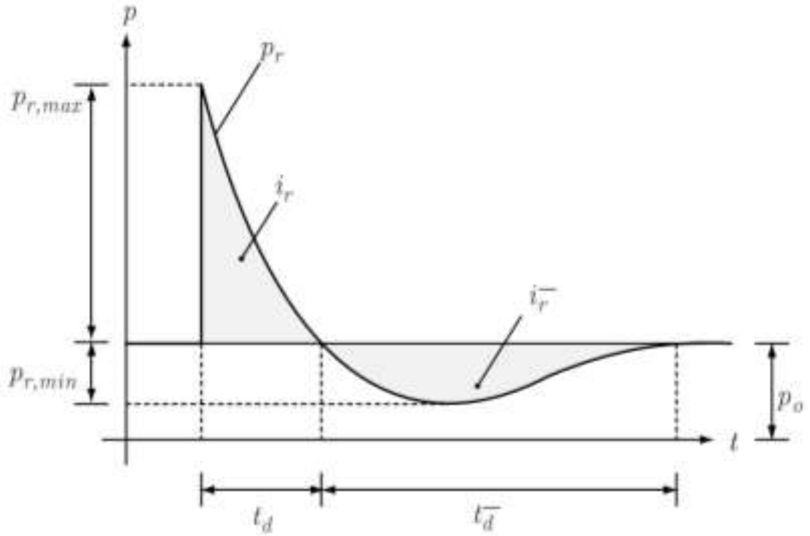

Figure 1. The idealization of wave explosion with time [3]

When $P_{r,max}$ is the peak of the reflected pressure, t_d is the positive phase duration, and b is the coefficient that describes the rate of decay of the pressure-time curve, which is known as the waveform or decay parameter. Following the positive phase comes a period of 'negative' pressure (below atmospheric), which is a partial vacuum caused by over-expansion of the shocked air. An idealized pressure-time profile for a reflected blast wave is shown in Figure 1, when $P_{r,min}$ is the peak under pressure, t_d^- is the negative phase duration, and the positive and negative phase impulses, i_r and i_r^-, are given as the temporal integral of the positive and the negative phase pressure.

1.2. Literature Review

Further research needs to be done related to the explosion load that has been done by modifying the Friedlander explosion load that has been included in research journals such as,

M. Teich in 2012, with Normative approach research that is reviewed as a Normative Equation (Positive Phase without negative phase) [4]. Rigby [2] states that there are four models, namely: Negative Bilinear, Extended Friedlander, Extended Friedlander Teich, and Linear-Cubic. Based on the book Infrasound Monitoring for Atmospheric Studies in Chapter 8, about Explosion Source Models [5], there are several equations, namely Granström negative and positive phase triangle, and Reed's 4th order of the polynomial equation. The equation of 6th order polynomial modifications that have been made needs to be evaluated, in terms of the approach to the Friedlander ideal equation, to make it easier to solve any problem in the numerical solution.

2. FRIEDLANDER BLAST WAVE

2.1. Positive Phase and Negative Phase

Teich and Gebbeken took a more general approach by conducting a parametric study on Elastic Single-Degree-of-Freedom (SDOF) systems to quantify the effect of the negative phase relative to the impact of the positive phase alone. It was found that for specific configurations of scaled distance and dynamic target properties, the inclusion of the negative phase in the model could cause midspan displacement of up to an order of magnitude higher than that of the positive phase [4].

In the 2019 research journal, the analysis of the effects of numerical detonation on the stiffness of concrete plate orthotropic, in the elastic range of the material, was mentioned. The effects of the stiffness configuration, the location of the localized blast load, and the effect of thickness on the vertical plate deflection were solved numerically by using two assistive equations in the x and y directions. The analysis produces a vertical deflection of the time relationship that can be used to determine the stress distribution and the maximum stress value of the plates. The effect of the explosion load on orthotropic plates is in the post-explosion or post-negative position, which is most influential on the magnitude of the deflection [6].

2.2. Comparison Between Modified Friedlander Equation and Friedlander 1946 Equation

2.2.1. Normative Equation (Linear Positive Phase)

The threat level of the United States GSA (2001) and ISO (2007) determines the blast load by determining the peaks that reflect excess pressure and cause impulses. The data shows that the calculations can be done with triangular images, as shown in the equation and figure below: [5]

$$\mathbf{P_{r0}^{lin}(t)} = \begin{cases} \widehat{\mathbf{P_{r0}}}\left(\mathbf{1} - \frac{\mathbf{t}}{\widehat{\mathbf{td}}}\right) & \mathbf{t} \le \widehat{\mathbf{td}} \\ \mathbf{0} & \mathbf{t} > \widehat{\mathbf{td}} \end{cases} \tag{2}$$

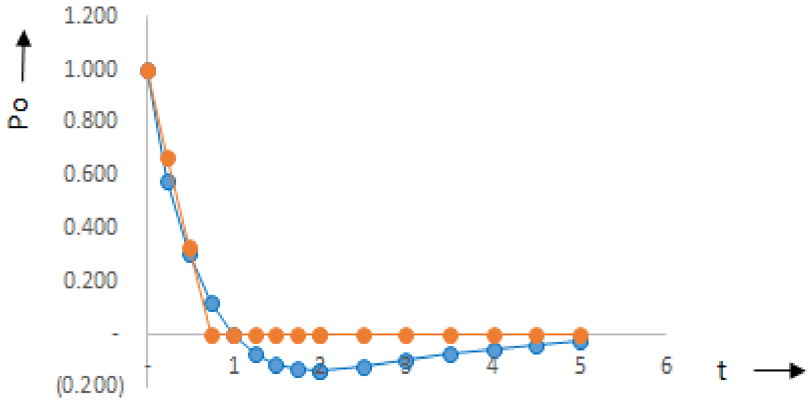

Figure 2. The comparison between normative equations and Friedlander 1946 equation (Source: analysis, 2019)

2.2.2. Equation of the Linear Positive and Negative Phase Approaches

One of the ways to model the blast pressure is to break the linear functions, such as the Krauthammer and Altenberg approach. The positive phase is estimated as a linear form that forms a triangle, with the duration of the linear load, T_d, lin, impulse load adjusted to the empirical method of the triangle load, i.e., T_d, lin = $2i_r$ / $p_{R,\ MAX}$. The negative phase is modeled as a bilinear approach with an increase in time equal to 1/4 of the duration of the negative phase. Corresponding to the positive phase is the negative linear duration of the phase, $t_{d-,\ lin} = 2i_r^- / p_{r,\ min}$. The negative phase starts at t_d, instead of $t_{d,\ lin}$, which giving a zero period of pressure between linear positive and negative phases [2]. The equation is displayed as follows:

$$\mathbf{P(t)} = \begin{cases} \mathbf{p_{r,max}\left(1-\dfrac{t}{t_{d,lin}}\right)\ ;t \le t_{d,lin}} \\ \mathbf{0\ ;\ t_{d,lin} < t \le t_d} \\ \mathbf{-p_{r,min}\left(\dfrac{t-td}{0.25t_{d,lin}^-}\right);\ td < t \le t_d + 0.25t_{d,lin}^-} \\ \mathbf{-p_{r,min}\left(1-\dfrac{t-\left(td-0.25t_{d,lin}^-\right)}{0.75t_{d,lin}^-}\right);\ t_d + 0.25t_{d,lin}^- < t \le t_d + t_{d,lin}^-} \end{cases} \quad (3)$$

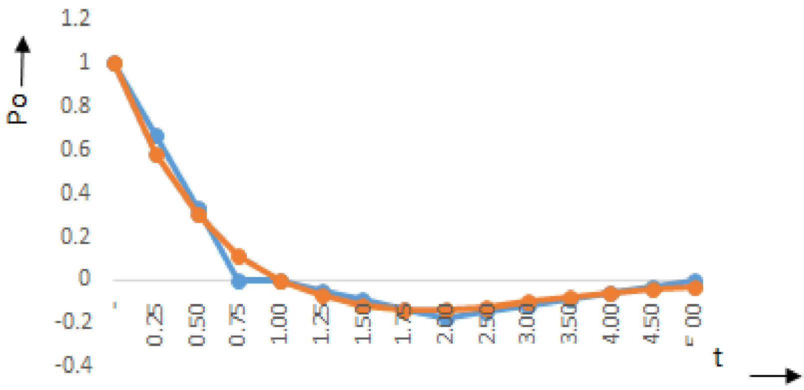

Figure 3. Comparison of Friedlander and Bilinier Negatives

(Source: Analysis, 2019)

2.2.3. Hybrid Granström – Friedlander Equation

Rigby et al. (2014) used a special case of G95 low explosive (G95 LE) phase deflagration for mounting a negative phase duration curve. The US Naval Facilities Engineering Command Design Manual (1986) and US Army Blast Effects Spreadsheet Design (2005) recommended the use of Link to Granström (1956) that is referred to as cubic negative phase expression by Rigby et al. (2014) and is used to construct hybrid phases with modified Friedlander for positive phases. [5]

However, the Friedlander phase with small waveform parameters is a positive triangle phase, which is much easier to evaluate and balance. It is a triangle approach that provides maximum positive moments for the shock phase, with a steep slope to the zero points towards the negative phase, which makes it possible to complete the maximum negative travel to get greater strength quickly. It eliminates waveform parameters and has a more interesting solution. The combination of a hybrid that adjusts to the slope and the impulse balance of the negative phase of Granström and the positive phase of the Friedlander triangle is stated as follows:

$$\boldsymbol{P^{+}(t) = P_r\left(1-\frac{t}{td}\right), \quad 0 \le t \le 1}$$

$$\boldsymbol{P^{-}(t) = P_r\frac{1}{6}\left(1-\frac{t}{td}\right)\left(1+\sqrt{6}-\frac{t}{td}\right)^2, \quad 1 < t \le 1+\sqrt{6}} \tag{4}$$

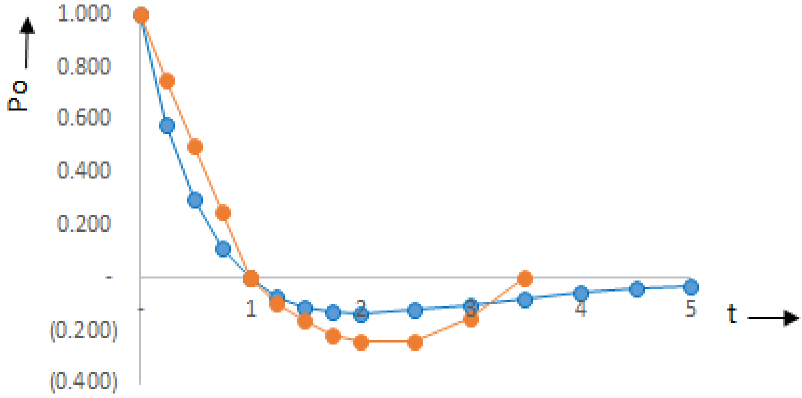

Figure 4. The Comparison Between Friedlander 1946 and Hybrid Granstrom

(Source: Analysis, 2019)

2.2.4. Extended Friedlander Equation

Another common method of modeling negative phases is to extend the boundary of Friedlander's equation (1) to t = ∞, which is the intersection of td. It is the approach that is adopted by Wei & Dharani, Gantes, and the LOAD_BLAST subroutine at LS-DYNA. Given the positive phase parameters of ConWep for pressure, duration, and impulses, parameters in the form of waves can be determined by integrating the Friedlander equation during the positive phase by making:

$$\mathbf{i_r{}^{+} = \int_0^{t_d} P_{r,max}\left(1-\frac{t}{t_d}\right)e^{-bt/td} = \frac{P_{r,max}t_d}{b^2}\left(b-1+e^{-b}\right)} \tag{5}$$

which can be solved iteratively to determine the value of b to give the correct impulse at a certain scale distance. As a negative phase, it can also be stated from the explanation that the shape of the negative phase is entirely dependent on the parameters of the positive phase. Because of the presence or absence of variables to control negative phase pressure and impulses, the values are given by using estimates that may not be following empirical equations. Integrating the negative phase of the Friedlander equation gives the following impulses.[2]

$$\mathbf{i_r}^- = \int_{t_d}^{\infty} \mathbf{P_{r,max}}\left(1-\frac{t}{t_d}\right)\mathbf{e^{-bt/td}} = \frac{\mathbf{P_{r,max}t_d}}{\mathbf{b^2}}\mathbf{e^{-b}} \tag{6}$$

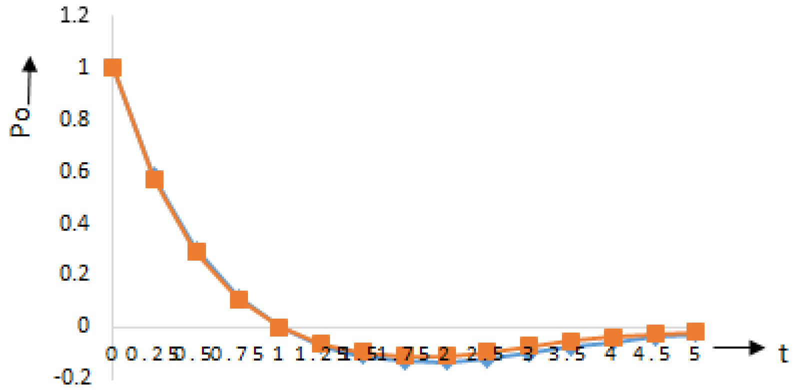

Figure 5. The Comparison Between Friedlander and Extended Friedlander

(Source: Analysis, 2019)

2.2.5. Extended Friedlander with Teich C- Equation

The value in the pressure peak in the Extended Friedlander equation can be more precise if it uses empirical values at certain scale distances. Equations in the positive and negative phases are separated as follows:

$$\mathbf{p_r(t)} = \begin{cases} \mathbf{P_{r,max}}\left(1-\frac{t}{t_d}\right)\mathbf{e^{-\alpha t/td}}\ ;\ \mathbf{t \le t_d} \\ \mathbf{C_r^- P_{so,max}}\left(1-\frac{t}{t_d}\right)\mathbf{e^{-\alpha t/td}}\ ;\mathbf{t \le t_d} \end{cases} \tag{7}$$

Although the introduction of the negative phase of the reflection coefficient makes it possible at the peak of the negative phase pressure to be controlled with the new waveform parameter, α, again given to match the positive impulse phase of the Friedlander equation with empirical predictions, for example, there is still no variable to control negative phase impulses. Extended Friedlander Teich provides negative phase impulses as follows. [2]

$$\mathbf{i_r}^- = \int_{t_d}^{\infty} \mathbf{C_r^- P_{so,max}}\left(1-\frac{t}{t_d}\right)\mathbf{e^{-\alpha t/td}} = \frac{\mathbf{C_r^- P_{so,max}t_d}}{\mathbf{\alpha^2}}\mathbf{e^{-\alpha}} \tag{8}$$

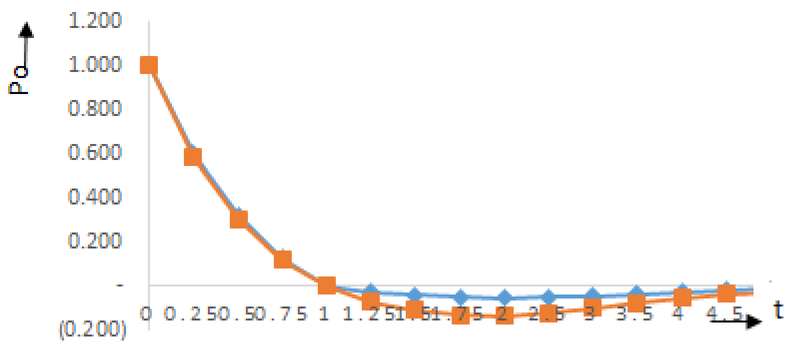

Figure 6. Friedlander Comparison with Extended of Teich C-
(Source: Analysis, 2019)

2.2.6. 4th Order of Reed's Polynomial Equations

Two pressure functions that consist of polynomials are introduced. The two functions are introduced as different group solutions, which are represented by M. Garces explosion, have a clear burst time, and are balanced against impulses. Reed's Equation 1977 states that the Friedlander explosion equation approach of 1946 is approximated by the 4th order polynomial model, which is stated as follows: [5]

$$\mathbf{P(t) = P_r\left(1-\frac{t}{td}\right)\left[1-\frac{7}{25}\frac{t}{td}\right]\left[1-\left(\frac{7}{25}\frac{t}{td}\right)^2\right],\qquad 0\le t\le \frac{25}{7}} \tag{9}$$

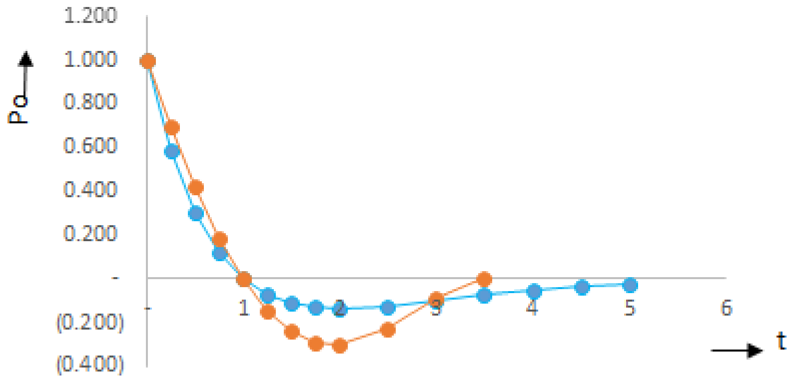

Figure 7. Friedlander's Comparison with Reed's (Source: Analysis, 2019)

3. MODIFICATION OF FRIEDLANDER'S EQUATION

3.1. Why Polynomial?

An overview of equations using polynomials needs to be done because polynomials are the simplest mathematical equations/functions, which only require multiplication and additional evaluation. Polynomials also have the flexibility to represent nonlinear relationships that are very common [7]. Two things why polynomial equations are used, first, because the equation minimizes errors, with a minimum of quadratic equations. Second, the equation reduces the worst error, which is at the time of the maximum equation [8]

3.2. Modification Using the Polynomial Approach

It is necessary to make modifications to the Friedlander equation approach to prove the statement about polynomials as the best equation. 6th Order polynomial equations are the best results from decreasing polynomial equations from 2nd to 7th order. The results of the polynomial of the 6th order show that it has fulfilled the ideal results in which the slope value is 0,00, the meaning is identical to the Friedlander 1946 equation.

$$\mathbf{P(t)=\begin{cases} P_{r,max}\left(0,00075\left(\frac{t}{td^+}\right)^6-0,0146\left(\frac{t}{td^+}\right)^5+0,1203\left(\frac{t}{td^+}\right)^4-0,5412\left(\frac{t}{td^+}\right)^3\right. \\ \left.+1,4095\left(\frac{t}{td^+}\right)^2-1,9752\left(\frac{t}{td^+}\right)+1\right) \qquad 0\le t\le td^+ \\ P_{r,min}\left(0,00075\left(\frac{t-td^+}{td^-}\right)^6-0,0146\left(\frac{t-td^+}{td^-}\right)^5+0,1203\left(\frac{t-td^+}{td^-}\right)^4-0,5412\left(\frac{t-td^+}{td^-}\right)^3\right. \\ \left.+1,4095\left(\frac{t-td^+}{td^-}\right)^2-1,9752\left(\frac{t-td^+}{td^-}\right)+1\right) \qquad td^+\le t\le (td^++td^-) \\ P_{r,min}(0) \qquad t\ge (td^++td^-) \end{cases}} \tag{10}$$

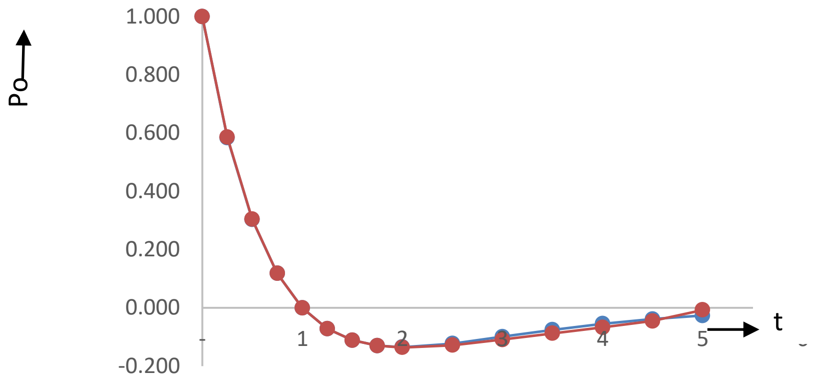

Figure 8. The Modification of Friedlander Equation with Order 6th Polynomial (Source: Analysis, 2019)

4. EVALUATION OF FRIEDLANDER'S MODIFICATIONS

All the existing equations evaluated based on the category of the ideal equation of the Friedlander 1946, with the following figure 9 and table 1.

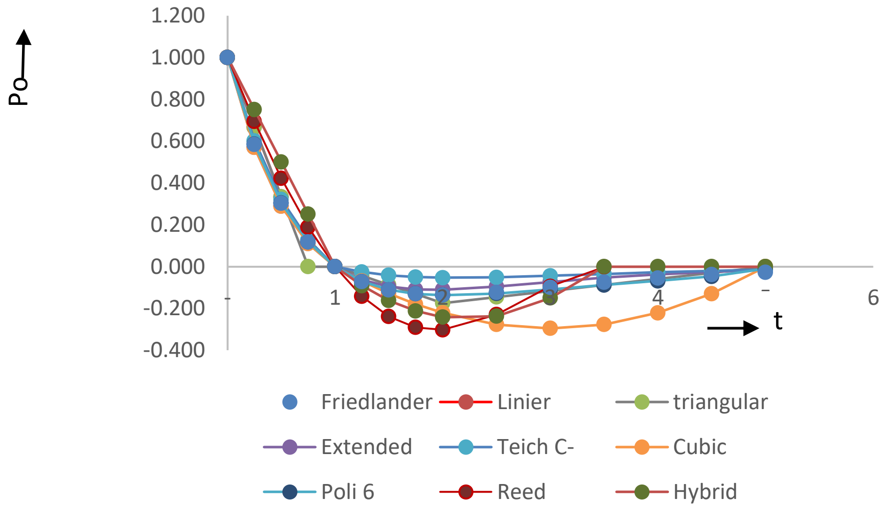

Figure 9. Friedlander Modified Chart Normal Condition (Source: Analysis, 2019)

Table 1. Evaluation of Modification Models of Friedlander (Source: Analysis, 2019)

No	Models	Quantitative Requirements	
		error	slope
1	Linear	28,36%	15,990
2	triangular	10,72%	5,967
3	Extended	5,82%	1,520
4	Teich C-	14,20%	10,065
5	Cubic	26,55%	9,343
6	Reed	29,18%	4,794
7	Hybrid	28,78%	1,928
12	Polynomial 6th	1,91%	0,005

It is necessary to make modifications to the Friedlander equation approach to make a statement about polynomials as the best equation. Based on the results of the 6th order polynomial, it shows that the polynomial of the 6th order has fulfilled the ideal results when the slope value is 0,005. The meaning is identical to the Friedlander 1946 equation.

5. CONCLUSION

The 6th order polynomial equation has the lowest error factor corresponding to the Friedlander exponential equation compared to the previous equations, namely: a) Bilinear Negative, b) Extended Friedlander, c) Extended Friedlander with Teich C-, d) Cubic Negative e) Normative, f) Cubic Linear and g) Reed. The 6th order polynomial equation approach for the Friedlander equation can be used as an explosion load in numerical solutions to obtain a dynamic response due to an explosion load.

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